Storage rings as detectors for relic gravitational–wave background ?

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Abstract

We argue that storage rings can be used for the detection of low–frequency gravitational–wave background. Proceeding from the measurements by Schin Daté and Noritaka Kumagai (Nucl. Instrum. Meth. **A421**, 417 (1999)) and Masaru Takao and Taihei Shimada (Proceedings of EPAC 2000, Vienna, 2000, p.1572) of variations of the machine circumference of the SPring–8 storage ring we explain the systematic shrinkage of the machine circumference by the influence of the relic gravitational–wave background. We give arguments against a possibility to explain the observed shrinkage of the machine circumference of the SPring–8 storage ring by diastrophic tectonic forces. We show that the forces, related to the *stiffness* of the physical structures, governing the path of the beam, can be neglected for the analysis of the shrinkage of the machine circumference caused by the relic gravitational–wave background. We show the shrinkage of the machine circumference can be explained by a relic gravitational–wave background even if it is treated as a stochastic system incoming on the plane of the machine circumference from all quarters of the Universe. We show that the rate of the shrinkage of the machine circumference does not depend on the radius of the storage ring and it should be universal for storage rings with any radii.

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1 Introduction

The existence of gravitational waves has been predicted by Einstein's general theory of relativity [1]–[3] . Starting with the pioneering work by Weber [4] one of the most challenging problems of experimental physics is the detection of gravitational radiation. In the seventies of the last century the existence of gravitational waves has been confirmed indirectly in a set of accurate measurements of secular orbital period changes in the Hulse–Taylor binary pulsar [5]. An attempt for the observation of the cosmic low–frequency gravitational–wave background has been undertaken by Stinebring, Ryba, Taylor and Romani [6].

An interesting influence of gravity on the parameters of storage rings of the Large Electron Positron Collider (LEP) at CERN and the SPring–8 in Japan has been found by Arnaudon *et al.* [7] and Daté and Kumagai [8], and Takao and Shimada [9], respectively. Below we will discuss only the measurements for the SPring–8 storage ring [8, 9], though our results should be applicable also to other storage rings.

In the analysis of the influence of gravity on the SPring–8 electron storage ring Daté, Kumagai, Takao and Shimada have considered the changes of the machine circumference $C_0 \simeq 1436\,\mathrm{m}$ in dependence of gravitational coupling of the storage ring to the Moon and the Sun. According to [8, 9], the change ΔC of the reference value of the machine circumference is defined by the gravitational interaction of the electron storage ring with the Moon and the Sun due to the tidal and seasonal forces. A total rate of a change of the machine circumference can be written as

$$\frac{\Delta C}{\Delta t} = \Big(\frac{\Delta C}{\Delta t}\Big)_{\rm m} + \Big(\frac{\Delta C}{\Delta t}\Big)_{\rm s} + \Big(\frac{\Delta C}{\Delta t}\Big)_{\rm us}, \tag{1.1}$$

where first two terms are caused by the tidal (m) and seasonal (s) forces, but the third term describes a rate of a change of the machine circumference due to unknown sources (us).

The theoretical predictions for $(\Delta C/\Delta t)_{\rm m} + (\Delta C/\Delta t)_{\rm s}$ caused by the tidal and seasonal forces have been fully confirmed experimentally [8, 9]. Nevertheless, measuring the rate $\Delta C/\Delta t$ of the changes of the machine circumference of the storage ring there has been found a systematic shrinkage of the machine circumference with the rate of about $2 \times 10^{-4}\,\mathrm{m/yr}$ [9], which cannot be explained by the tidal and seasonal forces induced by the Moon and the Sun. In (1.1) this shrinkage is described by the third term $(\Delta C/\Delta t)_{\rm us}$. In this letter we give arguments that this phenomenon can be understood as an influence of a cosmic very low–frequency gravitational–wave background. Therefore, below we denote the third term as $(\Delta C/\Delta t)_{\rm gw}$.

The paper is organized as follows. In Section 2 we estimate the rate of the shrinkage of the machine circumference due to the gravitational strain. In Section 3 we solve the equations of motion of the storage ring in the field of the cylindrical relic gravitational wave. We show that the solution of the equations of motion gives the same result obtained within the hypothesis of the *gravitational strain*. We show that the rate of the shrinkage of the machine circumference does not depend on the radius of the storage ring and should be universal for storage rings with any radii. In Section 4 we give arguments against a possibility to explain the observed shrinkage of the machine circumference of the SPring–8 storage ring by diastrophic tectonic forces. In Section 5 we discuss the influence of the

stiffness of the physical structures of the storage ring, governing the path of the beam. We argue that the forces, induced by the *stiffness* of the physical structures of the storage ring, governing the path of the beam, can be neglected for the analysis of the shrinkage of the machine circumference caused by the relic gravitational–wave background. In Section 6 we investigate the shrinkage of the machine circumference induced by a stochastic spherical relic gravitational–wave background incoming on the plane of the machine circumference from all quarters of the Universe. We show that the stochastic relic gravitational–wave background, incoming on the plane of the machine circumference from all quarters of the Universe, does not destroy the shrinkage of the machine circumference. The former is due to the fact that the effect of the shrinkage of the machine circumference is of the second order in gravitational wave interactions. We show that the independence of the rate of the shrinkage of the machine circumference on the radius of the storage ring retains in the case of the interaction of the storage ring with the stochastic relic gravitational–wave background. In the Conclusion we discuss the obtained results.

2 Gravitational strain and shrinkage of machine circumference

It is well–known that on the Earth one of the main fundamental effects of gravitational waves is the *gravitational strain*: a fractional distortion in the length of the object induced by the gravitational field [3].

In this connection we assume that the storage ring is sensitive to the influence of low–frequency gravitational waves, which produce a variation $\delta C_{\rm gw}(t)$ of the machine circumference C_0. Following [1]–[3] we treat low–frequency gravitational waves as perturbations of the metric.

For the calculation of $\delta C_{\rm gw}(t)$ we define a perturbation of the metric $h_{ab}(t,z)\,(a,b = x,y,z)$ as a monochromatic plane wave traveling along the z–axis with frequency ω and wave number $k = \omega/c$ [1]–[3]

$$h_{ab}(t,z) = \left(\begin{array}{ccc} h_{xx}(t,z) & h_{xy}(t,z) & 0 \\ h_{yx}(t,z) & h_{yy}(t,z) & 0 \\ 0 & 0 & 0 \end{array}\right) = \left(\begin{array}{ccc} \Delta_+ & \Delta_\times & 0 \\ \Delta_\times & -\Delta_+ & 0 \\ 0 & 0 & 0 \end{array}\right)\cos(\omega t - kz + \delta), \quad (2.1)$$

where Δ_+ and $\Delta_\times$ are constant amplitudes of the diagonal and non–diagonal components of the monochromatic plane wave, $h_{tt}(t,z) = h_{ta}(t,z) = h_{at}(t,z) = 0$ [1]–[3] and δ is an arbitrary phase. We define the monochromatic plane wave in the so–called *transverse traceless* gauge $h_{aa}(t,z) = h_{xx}(t,z) + h_{yy}(t,z) = 0$ (see pp.946–948 of Ref.[1]).

Placing the storage ring in the xy–plane at $z = 0$ the variation $\delta C_{\rm gw}(t)$ can be defined by the contour integral

$$\delta C_{\rm gw}(t) = \oint_{C_0}\sqrt{dx^2 + dy^2 + h_{xx}(t,0)dx^2 + h_{yy}(t,0)dy^2 + 2h_{xy}(t,0)dxdy} - C_0 =$$
$$= \frac{C_0}{2\pi}\int^{2\pi}_0 (\sqrt{1 - h_{xy}(t,0)\sin 2\varphi - h_{xx}(t,0)\cos 2\varphi} - 1)\,d\varphi = \oint_{C_0}\delta\ell_{\rm gw}, \quad (2.2)$$

where we have used polar coordinates $x = (C_0/2\pi)\cos\varphi$ and $y = (C_0/2\pi)\sin\varphi$ and the relation $h_{yy}(t,0) = -h_{xx}(t,0)$ (2.1). A change of the length of a segment between two

adjacent points of the machine circumference of the storage ring due to the *gravitational strain* caused by the monochromatic plane wave $h_{ab}(t,z)$ we denote as $\delta\ell_{\rm gw}$. Expanding the square root in powers of $h_{ab}(t,0)$ we represent $\delta\ell_{\rm gw}$ in the following form

$$\delta\ell_{\rm gw} = \frac{C_0}{2\pi}\Big(-\frac{1}{2}(h_{xx}\cos 2\varphi + h_{xy}\sin 2\varphi) - \frac{1}{8}(h_{xx}\cos 2\varphi + h_{xy}\sin 2\varphi)^2 + \ldots\Big)d\varphi. \quad (2.3)$$

Keeping the first non–vanishing contribution we get

$$\delta C_{\rm gw}(t) = \oint_{C_0}\delta\ell_{\rm gw} \simeq -\frac{1}{16}\,C_0\,(h^2_{xx}(t,0) + h^2_{xy}(t,0)) = -\frac{1}{16}\,C_0\,h^2_0\,\cos^2(\omega t + \delta), \quad (2.4)$$

where the amplitude h_0 is equal to $h_0 = \sqrt{\Delta^2_+ + \Delta^2_\times}$.

We would like to accentuate that the amplitude h_0 of the monochromatic plane wave is not the real amplitude of the relic gravitational–wave background. The relation of the amplitude h_0 to the amplitude $h^{\rm gw}_0$ of the relic gravitational–wave background can be found, for example, in the following way.

Notice that a real relic gravitational–wave background should be treated as a perturbation of the Friedmann–Robertson–Walker metric [1]–[3]. In terms of a perturbation of the Friedmann–Robertson–Walker metric, caused by the relic gravitational–wave background $h^{\rm gw}_{ab}(t,z)$, a change of the length of a segment between two adjacent points of the machine circumference of the storage ring can be determined by [1]–[3]

$$\delta s_{\rm gw} = R_{\rm U}\Big(-\frac{1}{2}(h^{\rm gw}_{xx}\cos 2\varphi + h^{\rm gw}_{xy}\sin 2\varphi) - \frac{1}{8}(h^{\rm gw}_{xx}\cos 2\varphi + h^{\rm gw}_{xy}\sin 2\varphi)^2 + \ldots\Big)d\varphi, \quad (2.5)$$

where $R_{\rm U}$ is the radius of the Universe at the present time. According to [1]–[3], the radius of the Universe is equal to $R_{\rm U} = (c/H_0)\sqrt{k/(\Omega - 1)}$, where $c = 9.45\times 10^{15}\,{\rm m}\cdot{\rm yr}^{-1}$, $H_0 = (7.63 \pm 0.75)\times 10^{-11}\,{\rm yr}^{-1}$ [10], Ω is a *density parameter* as the ratio of the energy density in the Universe to the critical energy density [1]–[3], and $k = 0, \pm 1$ for flat, closed and open Universe, respectively [1]–[3]: (1) $k = 0$ with $\Omega = 1$, (2) $k = 1$ with $\Omega > 1$ and (3) $k = -1$ with $\Omega < 1$. For our estimate we will use $R_{\rm U} \sim c/H_0 = 1.25\times 10^{26}\,{\rm m}$. This agrees with the value of the *Volume today* equal to $V = 2\pi^2 R^3_{\rm U} = 3.83\times 10^{79}\,{\rm m}^3$ (see [1], p.738, Box 27.4).

It is obvious that the contour integral of $\delta s_{\rm gw}$ over the machine circumference of the storage ring should give the same variation of the length of the machine circumference as Eq.(2.4):

$$\delta C_{\rm gw}(t) = \oint_{C_0}\delta\ell_{\rm gw} = \oint_{C_0}\delta s_{\rm gw}. \quad (2.6)$$

Keeping the first non–vanishing contributions we obtain the relation between h_0 and $h^{\rm gw}_0$ equal to

$$h^{\rm gw}_0 = \sqrt{\frac{C_0}{2\pi R_{\rm U}}}\,h_0 \sim 1.4\times 10^{-12}\,h_0, \quad (2.7)$$

where $h^{\rm gw}_0 = \sqrt{(\Delta^{\rm gw}_+)^2 + (\Delta^{\rm gw}_\times)^2}$.

Since by definition of a perturbation, $h_0 \ll 1$, the relation (2.7) gives a correct upper limit on the value of the real amplitude of the gravitational–wave background $h^{\rm gw}_0 \ll$

1.4×10^{-12} [1]–[3]. A more detailed estimate for $h_0^{\rm gw}$, related to the experimental shrinkage of the machine circumference of the storage ring [9], we derive below.

Notice that it is rather clear that the contribution of the gravitational waves to the variation of the machine circumference, $\delta C_{\rm gw}(t) \sim O((h_0^2)$, is of the second order. In fact, the mass quadrupole moment of the storage ring, located in the xy–plane at $z = 0$, has only two equal components $D_{xx} = D_{yy} = D$. Due to this, the interaction of this mass quadrupole moment with gravitational waves is proportional to $D\,(h_{xx} + h_{yy})$, which is zero by definition for gravitational waves in the *transverse traceless gauge* $h_{xx} = -h_{yy}$ [1]–[3].

The change of the storage ring of the machine circumference $\Delta C_{\rm gw}$ induced by the gravitational waves (2.1) for the time interval $\Delta t = t_2 - t_1$ is equal to

$$\Delta C_{\rm gw} = \delta C_{\rm gw}(t_2) - \delta C_{\rm gw}(t_1) = \frac{1}{16}\,C_0\,h_0^2\,\sin(\omega\Delta t)\,\sin(\omega(t_2+t_1)+2\delta). \tag{2.8}$$

For the rate of the change of the machine circumference at $\Delta t \to 0$ we get

$$\frac{\Delta C_{\rm gw}(t)}{\Delta t} = \frac{1}{16}\,C_0\,h_0^2\,\omega\,\sin(2\omega t + 2\delta). \tag{2.9}$$

For the comparison with the experimental rate we have to average the theoretical rate (2.9) over the data–taking period τ. This gives

$$\Big\langle\frac{\Delta C_{\rm gw}(t)}{\Delta t}\Big\rangle_{\tau} = \frac{1}{16}\,C_0\,h_0^2\,\omega\,\frac{1}{\tau}\int^{+\tau/2}_{-\tau/2}dt\,\sin(2\omega t + 2\delta) = \frac{1}{16}\,C_0\,h_0^2\,\sin 2\delta\,\frac{\sin\omega\tau}{\tau}. \tag{2.10}$$

In the low–frequency limit $\omega\tau \ll 1$, corresponding to the case of the relic gravitational–wave background, the relation (2.8) can be transcribed into the form

$$\frac{1}{C_0}\Big\langle\frac{\Delta C_{\rm gw}(t)}{\Delta t}\Big\rangle_{\tau} = \frac{1}{16}\,h_0^2\,\omega\,\sin 2\delta. \tag{2.11}$$

Since the experimental rate of the change of the machine circumference is equal to

$$\frac{1}{C_0}\Big(\frac{\Delta C}{\Delta t}\Big)_{\rm exp} \simeq -1.4\times 10^{-7}\,{\rm yr}^{-1}, \tag{2.12}$$

a comparison of theoretical and experimental rates leads to the relation

$$\omega\,h_0^2\,\sin 2\delta = \frac{16}{C_0}\,\Big(\frac{\Delta C}{\Delta t}\Big)_{\rm exp} \simeq -\,2.2\times 10^{-6}\,{\rm yr}^{-1}. \tag{2.13}$$

The experimentally observed shrinkage of the machine circumference of the storage ring [9] imposes a constraint on the phase of the gravitational wave, i.e. $-\sin 2\delta > 0$. For further estimates we set $|\sin 2\delta| \sim 1$.

Then, since $h_0 \ll 1$, we get the lower limit on the frequency, $\omega \gg 2\times 10^{-6}{\rm yr}^{-1}$. This corresponds to an oscillation period $T \ll 3\times 10^{-3}\,{\rm Gyr}$ of the shrinkage of the machine circumference, which is smaller compared with the age of the Universe $T \simeq 15\,{\rm Gyr}$ [11]. Since the oscillation period exceeds greatly any reasonable interval of experimental measurements, the rate of the shrinkage of the machine circumference, induced by the

relic gravitational–wave background, should be constant in time during any data–taking period. This agrees with Eq.(2.11).

We can also give a lower limit on the amplitude h_0. According to the experimental data by Takao and Shimada [9], the oscillation period of the rate of the machine circumference (2.12) should be much greater than 5 years, $T \gg 5\,\mathrm{yr}$. This gives $\omega \ll 1\,\mathrm{yr}^{-1}$ and $h_0 \gg 10^{-3}$ and according to (2.7) we get $h^{\rm gw}_0 \gg 10^{-15}$, the lower limit on the amplitude of the relic gravitational–wave background imposed by the experimental shrinkage of the machine circumference of the storage ring (2.12).

The rate of the shrinkage of the machine circumference, represented in terms of the relic gravitational–wave perturbations of the Friedmann–Robertson–Walker metric (2.5), reads

$$\Big\langle\frac{\Delta C_{\rm gw}(t)}{\Delta t}\Big\rangle_{\tau} = \frac{\pi}{8}\,R_{\rm U}\,(h^{\rm gw}_0)^2\,\omega\,\sin 2\delta. \tag{2.14}$$

This shows that the rate of the shrinkage of the machine circumference does not depend on the length of the machine circumference of the storage ring.

Thus, in our interpretation of the shrinkage of the machine circumference as induced by the relic gravitational–wave background, the value of the rate of the shrinkage of the machine circumference should be universal and equal to $(\Delta C_{\rm gw}(t)/\Delta t)_{\rm exp} = -\,2\times 10^{-4}\,\mathrm{m/yr}$ [9] for storage rings with any radii both for the SPring–8 with radius $R_0 \simeq 229\,\mathrm{m}$ and for the DAPHNE with radius $R_0 \simeq 15\,\mathrm{m}$.

Another important quantity characterizing the relic gravitational–wave background is the density parameter $\Omega_{\rm gw}$ defined by [1]–[3]

$$\Omega_{\rm gw} = \frac{\omega^2 (h^{\rm gw}_0)^2}{12 H^2_0}. \tag{2.15}$$

For the frequency $\omega \ll 1\,\mathrm{yr}^{-1}$ we get the upper limit $\Omega_{\rm gw} \ll 10^{-10}$, where we have used that $\omega h_0 \sim 1.5\times 10^{-3}\sqrt{\omega} \ll 1.5\times 10^{-3}\,\mathrm{yr}^{-1}$ for $\omega \ll 1\,\mathrm{yr}^{-1}$ giving due to (2.7) the relation $\omega h^{\rm gw}_0 \sim 2\times 10^{-15}\sqrt{\omega} \ll 2\times 10^{-15}\,\mathrm{yr}^{-1}$. The estimate $\Omega_{\rm gw} \ll 10^{-10}$ does not contradict contemporary cosmological models [3].

3 Equations of motion and shrinkage of machine circumference

In this Section we show that the analysis of the influence of the relic gravitational–wave background through the solution of equations of motion for the storage ring in the cylindrical relic gravitational–wave field gives the same result that we have obtained in Section 2.

According to [1] (see pp.1004–1011 of Ref.[1]) the non–relativistic motion of a massive particle in the xy–plane at $z = 0$ induced by the cylindrical gravitational–wave background can be described the equations of motion

$$\begin{aligned}\frac{d^2x}{dt^2} &= -R^x_{0x0}\,x - R^x_{0y0}\,y,\\ \frac{d^2y}{dt^2} &= -R^y_{0x0}\,x - R^y_{0y0}\,y,\end{aligned} \tag{3.1}$$

where $R^{\alpha}_{\beta\gamma\delta}$ is the Riemann tensor defined by [1]

$$R^{\alpha}_{\beta\gamma\delta} = \frac{\partial \Gamma^{\alpha}_{\beta\delta}}{\partial x^{\gamma}} - \frac{\partial \Gamma^{\alpha}_{\beta\gamma}}{\partial x^{\delta}} + \Gamma^{\alpha}_{\mu\gamma}\,\Gamma^{\mu}_{\beta\delta} - \Gamma^{\alpha}_{\mu\delta}\,\Gamma^{\mu}_{\beta\gamma}. \tag{3.2}$$

The Christoffel symbols or differently the "covariant connection coefficients" $\Gamma^{\alpha}_{\lambda\mu}$ are determined in terms of the metric tensor [1] (see also [10])

$$\Gamma^{\alpha}_{\lambda\mu} = \frac{1}{2}\, g^{\alpha\nu}\Big(\frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\lambda}}{\partial x^{\nu}}\Big). \tag{3.3}$$

For the calculation of the Christoffel symbols we use the following metric tensor [1] (see also [10])

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(t - z), \tag{3.4}$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ and $h_{\mu\nu}(t-z)$ is a symmetric tensor defined in the *transverse traceless gauge* with non–zero components $h_{xx}(t - z) = -h_{yy}(t - z)$ and $h_{xy}(t - z) = h_{yx}(t - z)$ [1]. In terms of $h_{\mu\nu}(t - z)$ the Christoffel symbols are given by

$$\Gamma^{\alpha}_{\lambda\mu} = \frac{1}{2}\,\eta^{\alpha\nu}\Big(\frac{\partial h_{\mu\nu}}{\partial x^{\lambda}} + \frac{\partial h_{\lambda\nu}}{\partial x^{\mu}} - \frac{\partial h_{\mu\lambda}}{\partial x^{\nu}}\Big) - \frac{1}{2}\,h^{\alpha\nu}\Big(\frac{\partial h_{\mu\nu}}{\partial x^{\lambda}} + \frac{\partial h_{\lambda\nu}}{\partial x^{\mu}} - \frac{\partial h_{\mu\lambda}}{\partial x^{\nu}}\Big). \tag{3.5}$$

The components of the Riemann tensor contributing to the equations of motion (3.1)

$$\begin{aligned} R^{x}_{0x0} &= -\frac{d\Gamma^{x}_{0x}}{dt} - \Gamma^{x}_{x0}\Gamma^{x}_{0x} - \Gamma^{x}_{y0}\Gamma^{y}_{0x} \quad , \quad R^{x}_{0y0} = -\frac{d\Gamma^{x}_{0y}}{dt} - \Gamma^{x}_{x0}\Gamma^{x}_{0y} - \Gamma^{x}_{y0}\Gamma^{y}_{0y}, \\ R^{y}_{0x0} &= -\frac{d\Gamma^{y}_{0x}}{dt} - \Gamma^{y}_{x0}\Gamma^{x}_{0x} - \Gamma^{y}_{y0}\Gamma^{y}_{0x} \quad , \quad R^{y}_{0y0} = -\frac{d\Gamma^{y}_{0y}}{dt} - \Gamma^{y}_{x0}\Gamma^{x}_{0y} - \Gamma^{y}_{y0}\Gamma^{y}_{0y}. \end{aligned} \tag{3.6}$$

The Christoffel symbols read

$$\begin{aligned} \Gamma^{x}_{0x} &= +\frac{1}{2}\frac{dh_{xx}}{dt} - \frac{1}{4}\frac{d}{dt}(h^2_{xx} + h^2_{xy}) \qquad , \ \Gamma^{y}_{0y} = -\frac{1}{2}\frac{dh_{xx}}{dt} - \frac{1}{4}\frac{d}{dt}(h^2_{xx} + h^2_{xy}), \\ \Gamma^{x}_{0y} &= +\frac{1}{2}\frac{dh_{xy}}{dt} - \frac{1}{2}\,h_{xx}h_{xy}\,\frac{d}{dt}\ell n\Big(\frac{h_{xy}}{h_{xx}}\Big) \ , \ \Gamma^{y}_{0x} = +\frac{1}{2}\frac{dh_{xy}}{dt} + \frac{1}{2}\,h_{xx}h_{xy}\,\frac{d}{dt}\ell n\Big(\frac{h_{xy}}{h_{xx}}\Big). \end{aligned} \tag{3.7}$$

For the monochromatic gravitational waves the ratio $h_{xy}/h_{xx} = \Delta_{\times}/\Delta_{+}$ is constant and the Christoffel symbols Γ^{x}_{0y} and Γ^{y}_{0x} are linear in h_{ab}.

For the calculation of the components of the Riemann tensor, defining the equations of motion (3.1), we keep also the terms of order $O(h^2_{ab})$ inclusively and obtain

$$\begin{aligned} R^{x}_{0x0} &= -\frac{1}{2}\frac{d^2h_{xx}}{dt^2} + \frac{1}{4}\frac{d^2}{dt^2}(h^2_{xx} + h^2_{xy}) - \frac{1}{4}\Big[\Big(\frac{dh_{xx}}{dt}\Big)^2 + \Big(\frac{dh_{xy}}{dt}\Big)^2\Big], \\ R^{x}_{0y0} &= R^{y}_{0x0} = -\frac{1}{2}\frac{d^2h_{xy}}{dt^2}, \\ R^{y}_{0y0} &= +\frac{1}{2}\frac{d^2h_{xx}}{dt^2} + \frac{1}{4}\frac{d^2}{dt^2}(h^2_{xx} + h^2_{xy}) - \frac{1}{4}\Big[\Big(\frac{dh_{xx}}{dt}\Big)^2 + \Big(\frac{dh_{xy}}{dt}\Big)^2\Big]. \end{aligned} \tag{3.8}$$

Substituting (3.8) in the equations of motion (3.1) we get

$$\begin{aligned} \ddot{x} &= \frac{1}{2}\,\ddot{h}_{xx}(t)\,x + \frac{1}{2}\,\ddot{h}_{xy}(t)\,y - \frac{1}{4}\,(\ddot{h^2}(t) - \dot{h}^2(t))\,x, \\ \ddot{y} &= \frac{1}{2}\,\ddot{h}_{xy}(t)\,x - \frac{1}{2}\,\ddot{h}_{xx}(t)\,y - \frac{1}{4}\,(\ddot{h^2}(t) - \dot{h}^2(t))\,y, \end{aligned} \tag{3.9}$$

where overdots stand for the derivative with respect to time. We have denoted $h^2 = h^2_{xx} + h^2_{xy}$ and $\dot{h}^2 = \dot{h}^2_{xx} + \dot{h}^2_{xy}$.

The equations of motion (3.9) can be treated as the Lagrange equations derived from the Lagrange function

$$\begin{aligned} L(t,x,y,\dot{x},\dot{y}) &= \frac{1}{2}\,(\dot{x}^2+\dot{y}^2) - \frac{1}{8}\,(\ddot{h^2}(t) - \dot{h}^2(t))\,(x^2+y^2) \\ &\quad + \frac{1}{4}\,\ddot{h}_{xx}(t)\,(x^2-y^2) + \frac{1}{2}\,\ddot{h}_{xy}(t)\,xy. \end{aligned} \tag{3.10}$$

In the polar coordinates $x = r\cos\Phi$ and $y = r\sin\Phi$ we get

$$\begin{aligned} L(t,r,\varphi,\dot{r},\dot{\Phi}) &= \frac{1}{2}\,\dot{r}^2 - \frac{1}{8}\,(\ddot{h^2}(t) - \dot{h}^2(t))\,r^2 \\ &\quad + \frac{1}{2}\,r^2\dot{\Phi}^2 + \frac{1}{4}\,r^2\,(\ddot{h}_{xx}(t)\,\cos 2\Phi + \ddot{h}_{xy}(t)\,\sin 2\Phi). \end{aligned} \tag{3.11}$$

Assuming that the radius r is almost constant we can factorize radial and angular degrees of freedom.

$$\begin{aligned} L(t,r,\varphi,\dot{r},\dot{\Phi}) &= \frac{1}{2}\,\dot{r}^2 - \frac{1}{8}\,(\ddot{h^2}(t) - \dot{h}^2(t))\,r^2 \\ &\quad + R_0^2\Big[\frac{1}{2}\,\dot{\varphi}^2 + \frac{1}{4}\,(\ddot{h}_{xx}(t)\,\cos 2\Phi + \ddot{h}_{xy}(t)\,\sin 2\Phi)\Big]. \end{aligned} \tag{3.12}$$

where R_0 is the radius of the machine circumference, $R_0 = C_0/2\pi$.

The equations of motion for the radius $r(t)$ and the azimuthal angle $\Phi(t)$ are equal to

$$\begin{aligned} \ddot{r}(t) &= -\frac{1}{4}\,(\ddot{h^2}(t) - \dot{h}^2(t))\,r(t), \\ \ddot{\Phi}(t) &= -\frac{1}{2}\,(\ddot{h}_{xx}(t)\,\sin 2\Phi(t) - \ddot{h}_{xy}(t)\,\cos 2\Phi(t)). \end{aligned} \tag{3.13}$$

Since $\Phi(t) \ll 1$, the solution reads

$$\begin{aligned} \Phi(t) &= \frac{1}{2}\,h_{xy}(t) = \frac{1}{2}\,\Delta_\times\,\cos(\omega t + \delta), \\ \dot{\Phi}(t) &= \frac{1}{2}\,\dot{h}_{xy}(t) = -\frac{1}{2}\,\Delta_\times\,\omega\,\sin(\omega t + \delta). \end{aligned} \tag{3.14}$$

For the frequencies of the gravitational wave background corresponding to the low–frequency limit $\omega \to 0$ we get

$$\dot{\Phi}(t) = -\frac{1}{2}\,\Delta_\times\,\omega\,\sin\delta. \tag{3.15}$$

This predicts the rotation of the machine circumference with a practically constant velocity in dependence on the polarization and phase of the gravitational wave background.

Assuming that $r(t)$ is a smooth function of t and replacing the radius $r(t)$ by $R_0 = C_0/2\pi$ in the r.h.s. of (3.13) we get

$$\frac{1}{R_0}\,\frac{dr(t)}{dt} = -\frac{1}{4}\int_0^t d\tau\,(\ddot{h^2}(\tau) - \dot{h}^2(\tau)) + D \tag{3.16}$$

where a constant D cancels all constant contributions to the r.h.s. of (3.16).

For the relic monochromatic cylindrical gravitational wave $h_{xx} = \Delta_+ \cos(\omega t + \delta)$ and $h_{xy} = \Delta_\times \cos(\omega t + \delta)$ the r.h.s. of (3.15) is equal to

$$\frac{1}{R_0}\frac{dr(t)}{dt} = \frac{1}{16}\,h_0^2\,\omega\,\sin(2\omega t + 2\delta) - \frac{1}{8}\,h_0^2\omega^2 t. \tag{3.17}$$

At leading order in the low–frequency limit $\omega \to 0$ we get

$$\frac{1}{C_0}\frac{dC(t)}{dt} = \frac{1}{16}\,h_0^2\,\omega\,\sin 2\delta. \tag{3.18}$$

This agrees fully with our result (2.11) obtained within the hypothesis of the *gravitational strain*.

Some similar formulas calculated in this section one can find in the paper by van Holten [12] devoted to the analysis of the cyclotron motion in a gravitational–wave background.

The fluctuations of the Friedmann–Roberson–Walker metric $h_{ab}(t-z)$ we define as [13]–[19]

$$ds^2 = a^2(t)(-dt^2 + dz^2 + dx^2 + dy^2 + h^{\rm gw}_{ab}(t-z)dx^a dx^b), \tag{3.19}$$

where $a(t)$ is a scale factor [1]–[3]. From (3.19) we obtain [14]

$$h_{ab}(t-z) = a^2(t)h^{\rm gw}_{ab}(t-z). \tag{3.20}$$

The explicit expression for the scale factor $a(t)$ depends on the epoch of the evolution of the Universe [1]–[3] (see also [14]–[18]). The relation (3.20) does not contradict our estimate (2.7).

4 Shrinkage of the machine circumference as diastrophism of the Earth crust

In this Section we give arguments against a possibility to explain the observed shrinkage of the machine circumference of the SPring–8 storage ring by diastrophic tectonic forces or *diastrophism of the Earth crust*.

There are major forces acting within the Earth crust. They can be forces of *compression*, *tension* or *shearing*. They may be directly due to plate tectonics or caused by more localized or regionalized stresses. When these forces actually deform parts of the crust, the resulting landforms produced are said to have been formed by *diastrophism*. Diastrophism can cause *uplifting*, *rifting*, *doming*, and *tilting* of regions of the Earth surface. However, the major forms of diastrophism are associated with either *folding* or *faulting*. When these forces actually deform parts of the crust, the resulting landforms produced are said to have been formed by *diastrophism* [20].

Since the shrinkage of the machine circumference can be identified to some extent with the deformation of the part of the Earth crust, an alternative source of the systematic shrinkage could be, in principle, caused by *diastrophism*. First let us estimate the value of the *part of the Earth crust* which is undergone by *diastrophism*. The area occupied by the

storage ring is equal to $S_0 = C_0^2/4\pi = 0.164\,\mathrm{km}^2$ with a radius $R_0 = C_0/2\pi = 0.229\,\mathrm{km}$. The deformation of this part of the Earth crust leads to the systematic shrinkage of the crust to the center of the machine circumference of the storage ring. Since neither *faulting* nor *tilting* can contribute to this shrinkage, so only *folding* of the Earth crust can be attracted to the explanation of this phenomenon.

Indeed, *folding* occurs when rocks buckle or fold due to horizontal or vertical pressure. They are shaped into an arch (called *anticline*) or a trough (called *syncline*), or they may override an adjacent fold [20].

However, the linear scale of the machine circumference of the storage ring R_0 is smaller compared with linear scales L of tectonic forces providing *moldings* of the Earth crust, which are of order of a few kilometers, and, correspondingly $L \gg R_0$. Of course, the Global Positioning System (GPS) admits a measuring of motion of some parts of the Earth crust with a velocity comeasurable with the rate of the shrinkage of the machine circumference of the storage ring which is of order $2 \times 10^{-4}\,\mathrm{m/yr}$ [20]. However, it is very unlikely that in such a seismic active country as Japan a center of tectonic forces, leading to the shrinkage of the machine circumference, would be localized with a great precision at the center of the machine circumference during more than 5 years.

Therefore, we can conclude that the observed shrinkage of the machine circumference of the SPring–8 storage ring, located in Japan, can be hardly caused by diastrophic tectonic forces. It seems extremely incredible that the diastrophic tectonic forces, discussed above, would really be able to produce a longer then few–year lasting folding of the Earth crust with the scale $D \simeq 0.458\,\mathrm{km}$ to the center of the machine circumference of the storage ring.

Thus, one can believe that the mechanism of the shrinkage of the machine circumference related to the gravitational–wave background seems to be more credible and probable with respect to any one caused by tectonic forces.

5 Shrinkage of the machine circumference and stiffness of the physical structures, governing the path of the beam

In this section we discuss the influence of the forces, related to the *stiffness* of the physical structures of the storage ring, governing the path of the beam (mounts of magnets, for instance). In fact, one can imagine that the forces, induced by the *stiffness* of the physical structures of the storage ring, can prevent the machine circumference of the storage ring from the shrinkage caused by the relic gravitational–wave background. If it is so this should mean that the observed shrinkage of the machine circumference of the storage ring cannot be explained by the influence of the relic gravitational–wave background. Below we show that the forces, related to the *stiffness* of the physical structures of the storage ring, can be neglected for the analysis of the shrinkage of the machine circumference, caused by the relic gravitational–wave background.

Let us denote the forces, caused by the *stiffness* of the physical structures of the storage ring, as $\vec{F}_{\rm stiff}$. The observation of the fluctuations of the machine circumference, induced by the tidal and seasonal forces [8, 9], assumes that the forces, produced by the *stiffness*

of the physical structures of the storage ring, are smaller compared with the tidal and seasonal forces.

Since the seasonal forces are smaller compared with the tidal forces but have been measured experimentally by the change of the machine circumference, it is obvious that the forces, induced by the *stiffness* of the physical structures of the storage ring, should be smaller compared with the seasonal forces.

This can be written in the form of the inequality

$$|\vec{F}_{\rm stiff}| \ll |\vec{F}_{\rm s}(\vec{r}\,)|, \tag{5.1}$$

where the force $\vec{F}_{\rm s}(\vec{r}\,)$ is defined by [8]

$$\vec{F}_{\rm s}(\vec{r}\,) = -\bigtriangledown\, U_{\rm s}(\vec{r}\,). \tag{5.2}$$

The potential $U_{\rm s}(\vec{r}\,)$, produced by the Sun, is given by [8]

$$U_{\rm s}(\vec{r}\,) = G_N M_{\odot}\Big(\frac{1}{|\vec{R}_s - \vec{r}\,|} - \frac{1}{R_s} - \frac{\vec{r}\cdot\vec{R}_s}{R^3_s}\Big), \tag{5.3}$$

where $G_N = 6.636\times 10^{4}\,{\rm m^3\,kg^{-1}\,yr^{-2}}$ [10], $M_{\odot} = 1.989\times 10^{30}\,{\rm kg}$ is the mass of the Sun, $R_s = 1.496\times 10^{11}\,{\rm m}$ is the distance between centers of the Sun and the Earth, $|\vec{r}| = R_{\oplus} = 6.378\times 10^{6}\,{\rm m}$ is the radius of the Earth.

The rate of the change of the machine circumference, caused by the tidal and seasonal forces is of order of $|\Delta C/\Delta t| = 4\times 10^{-4}\,{\rm m/yr}$ [8, 9]. The the experimental rate of the shrinkage of the machine circumference, $|\Delta C/\Delta t| = 2\times 10^{-4}\,{\rm m/yr}$ [9], is of the same order of magnitude. This implies that the forces, leading to the shrinkage of the machine circumference, can be of gravitational nature. Moreover, the forces, induced by the *stiffness* of the physical structures of the storage ring, governing the path of the beam, should be smaller compared with the forces responsible for the shrinkage.

In order to get a quantitative confirmation of this assertion we suggest to compare the energy densities of the seasonal forces and the relic gravitational–wave background. Following [1]–[3] we define the energy density of the seasonal forces and the gravitational–wave background as

$$\begin{aligned} T^{\rm s}_{00} &\sim \frac{1}{32\pi c^2 G_N}\Big\langle\Big(\frac{2\pi}{T_s}\,U_{\rm s}(\vec{r}\,)\Big)^2\Big\rangle = \frac{\pi G_N M^2_{\odot} R^4_{\otimes}}{40 c^2 T^2_s R^6_s},\\ T^{\rm gw}_{00} &= \frac{c^2\omega^2 (h^{\rm gw}_0)^2}{32\pi G_N}, \end{aligned} \tag{5.4}$$

where $T_s = 0.5\,{\rm yr}$ is a period of the seasonal forces [1].

Assuming that the energy density of the gravitational–wave background $T^{\rm gw}_{00}$ should be of the same order of magnitude as the energy density of the seasonal forces $T^{\rm s}_{00}$, $T^{\rm gw}_{00} \sim T^{\rm s}_{00}$, we can get a constraint on the amplitude and frequency of the gravitational–wave background responsible for the observed shrinkage of the machine circumference [9]. Setting

[1]We have used for the estimate of the time–derivative of the potential $U_s(\vec{r}\,)$ the relation $|\dot{U}_{\rm s}(\vec{r}\,)| \sim \omega_s |U_{\rm s}(\vec{r}\,)|$, where $\omega_s = 2\pi/T_s$ is a characteristic frequency of the time–variations of the seasonal forces [21].

$T_{00}^{\rm gw} \sim T_{00}^{\rm s}$ we obtain

$$\omega h_0^{\rm gw} \sim \frac{2\pi}{\sqrt{5}}\,\frac{G_N M_{\odot} R_{\otimes}^2}{c^2 T_s R_s^3} \simeq 10^{-16}\,{\rm yr}^{-1}. \tag{5.5}$$

For $\omega \ll 1\,{\rm yr}^{-1}$ the relation (5.5) gives $h_0^{\rm gw} \gg 10^{-16}$ that agrees with our estimate $h_0^{\rm gw} \gg 10^{-15}$ given in section 2.

This should testify that the *stiffness* of the physical structures of the storage ring, governing the path of the beam, can be neglected for the analysis of the shrinkage of the machine circumference, caused by the relic gravitational–wave background.

6 Stochastic relic gravitational–wave background

In this section we analyse the shrinkage of the machine circumference [9] coupled to the relic gravitational–wave background treated as a stochastic system [14, 18, 22]–[24]. We show that the suggested explanation of the shrinkage of the machine circumference, measured at the SPring–8 [9], by the relic gravitational–wave background survives even if the storage ring interacts with the stochastic relic gravitational waves coming from all quarters of the Universe. This is related to the fact that the observed shrinkage of the machine circumference of the storage ring is an effect of the second order of the interaction of the gravitational waves with the machine circumference.

Below we consider spherical relic gravitational waves [24, 25] converging to the center of the machine circumference [2]. The relic gravitational waves are polarized in the $(\varphi_s\vartheta_s)$ plane, defined by unit vectors $\vec{e}_{\varphi_s}$ and $\vec{e}_{\vartheta_s}$ as it is shown in Fig.1, perpendicular to the direction of the propagation, which is anti–parallel to the unit vector $\vec{e}_r$. The polarization tensor, determined in the *transverse traceless gauge*, has the following non–vanishing components: $\Delta_{\varphi_s\varphi_s} = -\Delta_{\vartheta_s\vartheta_s} = \Delta_+$ and $\Delta_{\varphi_s\vartheta_s} = \Delta_{\vartheta_s\varphi_s} = \Delta_\times$.

It is convenient to analyse the influence of the stochastic relic gravitational–wave background on the shrinkage of the machine circumference in terms of the *gravitational strain* as it is done in Section 2.

The *gravitational strain* of the machine circumference, induced by the stochastic relic gravitational waves incoming from all quarters of the Universe, can be defined by

$$\delta C_{\rm gw}(t) = -\frac{C_0}{16\pi}\int d\Omega_s \int_0^{2\pi}\langle (h_{xx}(t)\,\cos 2\varphi + h_{xy}(t)\sin 2\varphi)^2\rangle d\varphi, \tag{6.1}$$

where $h_{xx}(t)$ and $h_{xy}(t)$ are given by

$$\begin{aligned} h_{xx}(t) &= \cos\vartheta_s\,(\Delta_+\,\cos 2\varphi_s + \Delta_\times\,\sin 2\varphi_s)\,\cos\Big(\omega\Big(t - \frac{R_0}{c}\Big) + \delta\Big), \\ h_{xy}(t) &= \cos\vartheta_s\,(-\Delta_+\,\sin 2\varphi_s + \Delta_\times\,\cos 2\varphi_s)\,\cos\Big(\omega\Big(t - \frac{R_0}{c}\Big) + \delta\Big) \end{aligned} \tag{6.2}$$

[2]The spherical gravitational waves, converging to the center of the machine circumference, we define as [25]: $h_{ab}(t, |\vec{r} - \vec{R}_0|) \sim \cos(\omega(t - |\vec{r} - \vec{R}_0|/c) + \delta)/|\vec{r} - \vec{R}_0|$, where the vector $\vec{R}_0$ is the radius–vector of the machine circumference, located in the plane of the machine circumference $|\vec{R}_0| = R_0 = C_0/2\pi$, and $\vec{r}$ is the radius–vector of the observer. It is zero , $\vec{r} = 0$, at the center of the machine circumference.

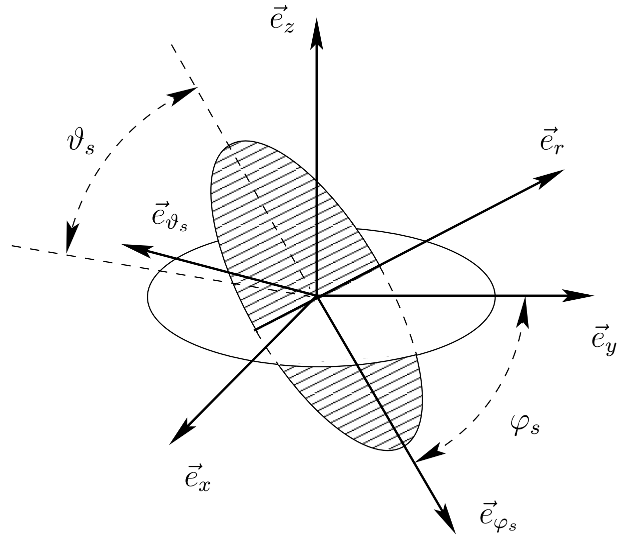

Figure 1: The orientation of the polarization plane of the stochastic relic gravitational waves relative to the plane of the machine circumference.

and $h_{yy}(t) = -h_{xx}(t)$. Below we neglect $R_0/c = 7.6 \times 10^{-7}\,\mathrm{s}$, where $R_0 \simeq 229\,\mathrm{m}$ is the radius of the machine circumference, relative to the data–taking period τ, which is about a few years [9]. The quantities $h_{xx}(t)$, $h_{yy}(t)$ and $h_{xy}(t)$ are the projections of the components of the polarization tensor of the spherical relic gravitational wave on the plane of the machine circumference (see Fig.1). They depend on the angles ϑ_s and φ_s, which are the angle of the slope of the polarization plane of the gravitational wave relative to the plane of the machine circumference and the azimuthal angle, respectively (see Fig.1). At $\vartheta_s = \varphi_s = 0$ we get a gravitational wave equivalent to the cylindrical gravitational wave defined by (2.1).

Integration over the angles ϑ_s and φ_s, where $d\Omega_s = \sin\vartheta_s d\vartheta_s d\varphi_s$, takes into account the contribution of the stochastic relic gravitational waves incoming on the plane of the machine circumference from all quarters of the Universe [3]. Following [18, 22]–[24] we assume that the stochastic relic gravitational–wave background is isotropic.

In Eq.(6.1) the brackets $\langle \ldots \rangle$ mean

$$\langle f \rangle = \int_0^\infty d\omega\, S_h(\omega)\, f(\omega), \tag{6.3}$$

where $S_h(\omega)$ is a spectral density, caused by the averaging over stochastic degrees of freedom of the relic gravitational–wave background [22]–[23]. We suppose that the spectral density $S_h(\omega)$ is normalized to unity.

The properties of the spectral density $S_h(\omega)$ depend on the theoretical model of the stochastic relic gravitational–wave background. We do not suggest any theoretic model of a stochastic gravitational–wave background and our approach to the description of the stochastic relic gravitational–wave background is phenomenological to full extent. The properties of the spectral density $S_h(\omega)$, such as a localization in the region of very low frequencies, $\omega \ll 1\,\mathrm{yr}^{-1}$, and so, we specify in terms of constraints on the averaged

[3] It is assumed that the Earth is transparent for the relic gravitational waves.

frequencies $\langle\omega\rangle$ and $\langle\omega^2\rangle$. These constraints come from the comparison of the experimental and theoretical rates of the shrinkage of the machine circumference, where the theoretical rate is defined by the interaction of the storage ring with the stochastic relic gravitational–wave background.

Substituting (6.2) into (6.1) we get

$$\begin{aligned}\delta C_{\rm gw}(t) &= -\frac{C_0}{16\pi}\int^{\infty}_0 d\omega\, S_h(\omega)\,\cos^2(\omega t+\delta)\int d\Omega_s\,\cos^2\vartheta_s\\ &\times \int^{2\pi}_0\Big[\Delta_+\,\cos 2(\varphi_s+\varphi)+\Delta_\times\,\sin 2(\varphi_s+\varphi)\Big]^2\,d\varphi,\end{aligned}\tag{6.4}$$

Integrating over the angular variables we obtain the *gravitational strain*, induced by the stochastic spherical relic gravitational–wave background incoming on the plane of the machine circumference from all quarters of the Universe. It reads

$$\delta C_{\rm gw}(t) = -\frac{4\pi}{3}\,\frac{1}{16}\,C_0 h^2_0\int^{\infty}_0 d\omega\, S_h(\omega)\,\cos^2(\omega t+\delta),\tag{6.5}$$

where $h_0=\sqrt{\Delta^2_+ +\Delta^2_\times}$.

The relative rate of the shrinkage of the machine circumference can be defined by

$$\frac{1}{C_0}\,\frac{\Delta C_{\rm gw}}{\Delta t}=\frac{4\pi}{3}\,\frac{1}{16}\,h^2_0\int^{\infty}_0 d\omega\,\omega\, S_h(\omega)\,\sin(2\omega t+2\delta).\tag{6.6}$$

In such a form the rate of the shrinkage of the machine circumference resembles two–point correlation functions of the operators of the gravitational waves appearing in the description of the relic gravitational–wave background as a stochastic system [22]–[24].

For the comparison of the theoretical rate of the change of the machine circumference (6.6) with the experimental data one has to average the theoretical rate over the data–taking period τ. This gives

$$\begin{aligned}\frac{1}{C_0}\Big\langle\frac{\Delta C_{\rm gw}}{\Delta t}\Big\rangle_\tau &= \frac{4\pi}{3}\,\frac{1}{16}\,h^2_0\int^{\infty}_0 d\omega\,\omega\, S_h(\omega)\frac{1}{\tau}\int^{+\tau/2}_{-\tau/2}dt\,\sin(2\omega t+2\delta)=\\ &= \frac{4\pi}{3}\,\frac{1}{16}\,h^2_0\,\sin 2\delta\int^{\infty}_0 d\omega\,\omega\, S_h(\omega)\,\frac{\sin\omega\tau}{\omega\tau}.\end{aligned}\tag{6.7}$$

Since we deal with a relic gravitational–wave background, we suppose that the frequencies of the relic gravitational waves satisfy the constraint $\omega\tau\ll 1$.

For the validity of this constraint we have to assume that the spectral density $S_h(\omega)$ is localized in the region of frequencies of order of $\omega\ll 1\,{\rm yr}^{-1}$.

In the case of the dominance of the region $\omega\tau\ll 1$ in the integrand of the integral over ω in the r.h.s. of (6.7), we can transcribe Eq.(6.7) into the form

$$\frac{1}{C_0}\Big\langle\frac{\Delta C_{\rm gw}}{\Delta t}\Big\rangle_\tau=\frac{4\pi}{3}\,\frac{1}{16}\,h^2_0\,\langle\omega\rangle\,\sin 2\delta,\tag{6.8}$$

where $\langle\omega\rangle$ we determine as an averaged frequency of the stochastic relic gravitational–wave background weighted with the spectral density $S_h(\omega)$. It reads

$$\langle\omega\rangle=\int^{\infty}_0 d\omega\,\omega\,S_h(\omega).\tag{6.9}$$

The expression (6.8) differs by a factor $4\pi/3$ from the rate of the shrinkage of the machine circumference given by Eq.(2.11). Such a factor is caused by the summation over all directions of the relic gravitational waves coupled to the storage ring. The appearance of the factor $4\pi/3$ does not change significantly our estimates made below Eq.(2.13).

Now from the comparison of the theoretical rate (6.8) with the experimental one (2.12) we get $\langle\omega\rangle \gg 5\times 10^{-7}\,\mathrm{yr}^{-1}$. For the averaged period of oscillations of the machine circumference this gives $\langle T\rangle \ll 10^{-2}\,\mathrm{Gyr}$.

The upper limit on the density parameter $\Omega_{\rm gw} \ll 10^{-10}$, given by (2.15), is left unchanged for $\sqrt{\langle\omega^2\rangle} \ll 1\,\mathrm{yr}^{-1}$, which is not related to the factor $4\pi/3$. Remind that the constraint $\sqrt{\langle\omega^2\rangle} \ll 1\,\mathrm{yr}^{-1}$ is caused by the experimental fact that the period of the observed shrinkage of the machine circumference of the SPring–8 storage ring should be greater than 5 years [9].

Of course, the constraints $\langle\omega\rangle \gg 5\times 10^{-7}\,\mathrm{yr}^{-1}$ and $\sqrt{\langle\omega^2\rangle} \ll 1\,\mathrm{yr}^{-1}$ can be justified only by the properties of the spectral density $S_h(\omega)$ within a certain theoretical model of the stochastic relic gravitational–wave background.

For the rate of the shrinkage of the machine circumference, given in terms of the stochastic relic gravitational–wave perturbations of the Friedmann–Robertson–Walker metric (2.5), we obtain

$$\Big\langle\frac{\Delta C_{\rm gw}}{\Delta t}\Big\rangle_{\tau} = \frac{4\pi}{3}\,\frac{\pi}{8}\,R_{\rm U}\,(h^{\rm gw}_0)^2\,\langle\omega\rangle\,\sin 2\delta, \tag{6.10}$$

where $h^{\rm gw}_0 = \sqrt{(\Delta^{\rm gw}_{+})^2 + (\Delta^{\rm gw}_{\times})^2}$.

The r.h.s. of (6.10) does not depend on the length of the the machine circumference. Therefore, the rate of the shrinkage of the machine circumference should be universal for all storage rings with any radii.

From the comparison of (6.10) with (6.8) one can conclude that the relation between the amplitudes h_0 and $h^{\rm gw}_0$, given by Eq.(2.7), is retained for the stochastic relic gravitational–wave background incoming on the plane of the machine circumference from all quarters of the Universe.

Thus, within our phenomenological approach to the description of the stochastic relic gravitational–wave background the interaction of the stochastic relic gravitational–wave background, incoming on the plane of the machine circumference from all quarters of the Universe, with the storage ring does not destroy the shrinkage of the machine circumference of the storage ring, observed in [9]. Formally, this is due to the phenomenon of the shrinkage of the machine circumference is of the second order in gravitational wave interactions.

7 Conclusion

The results obtained above should be understood as a hint that experimental analysis of fine variations of the machine circumferences of the storage rings can, in principle, contain an information about the relic gravitational–wave background on the same footing as the storage rings are sensitive to the tidal and seasonal forces [7]–[9].

We argue that if the systematic shrinkage of the machine circumference of the storage ring, observed at the SPring–8 [9], is caused by the influence of the relic gravitational–wave background, the same effect should be measured for the machine circumference of

the storage ring of any accelerator, for example, the LEP at CERN [7], the ELSA at University of Bonn, the DAPHNE at Frascati, the VEPP–4 at Novosibirsk and others.

We have shown that the rate of the shrinkage of the machine circumference, represented in terms of the relic gravitational–wave perturbations of the Friedmann–Robertson–Walker metric, does not depend on the length of the machine circumference and should be universal for any storage ring with any radius (see Eqs.(2.14) and (6.10)).

This makes very simple the experimental analysis of the validity of our hypothesis of the influence of the relic gravitational–wave background on the shrinkage of the machine circumference of the SPring–8 storage ring. Indeed, it is sufficient to measure the rates of the shrinkage of the machine circumferences of the storage rings of the LEP at CERN, the DAPHNE at Frascati, the VEPP–4 at Novosibirsk or of any other accelerators. If the rates of the shrinkage of the machine circumferences of the storage rings would have been found comeasurable with the value $(\Delta C(t)/\Delta t)_{\rm exp} = -\,2\times 10^{-4}\,{\rm m/yr}$, obtained for the SPring–8 storage ring [9], this should testify the detection of the relic gravitational–wave background. Any negative result should bury the hypothesis.

We argue that the shrinkage of the machine circumference of the storage ring cannot be related to diastrophic tectonic forces. Then, since the value of the rate of the shrinkage of the machine circumference is comeasurable with the change of the machine circumference, induced by the seasonal forces, the influence of the *stiffness* of the physical structures of the storage ring, governing the path of the beam (mounts of magnets, for instance), can be neglected. In fact, as has been measured by Daté and Kumagai [8] and Takao and Shimada [9] the forces, related to the *stiffness* of the physical structures of the storage ring, governing the path of the beam (mounts of magnets, for instance), are smaller compared with the seasonal forces.

We have solved Einstein's equations of motion for the storage ring in the field of the cylindrical relic gravitational wave and computed the rate of the shrinkage of the machine circumference. We have shown that the rate of the shrinkage of the machine circumference, obtained from the solution of Einstein's equations of motion for the storage ring in the field of the cylindrical relic gravitational wave, coincides with the rate obtained in terms of the *gravitational strain*. In addition to the shrinkage of the machine circumference we have found a slow rotation of the storage ring defined by the non–diagonal component of the polarization tensor of the relic gravitational wave in the *transverse traceless gauge*, $\Phi(t) = h_{xy}(t)/2$,

Finally we have discussed the interaction of the storage ring with a stochastic relic spherical gravitational–wave background. We have shown that, since the shrinkage of the machine circumference is a phenomenon of the second order in gravitational wave interactions, it cannot be destroyed even if one takes into account the contribution of the relic gravitational waves incoming on the plane of the machine circumference from all quarters of the Universe. We have obtained an additional factor $4\pi/3$ relative to the rate of the shrinkage of the machine circumference, induced by the cylindrical relic gravitational–wave background (2.11) [4]. This changes only the lower limit of the frequencies of the gravitational waves responsible for the observed shrinkage.

[4]Of course, the stochastic relic gravitational–wave background would not produce a rotation of the machine circumference, which is linear in the gravitational wave $\Phi(t) = h_{xy}(t)/2$. Such a rotation has been obtained in Section 3 by solving Einstein's equations of motion of the storage ring in the field of the cylindrical relic gravitational wave.

Indeed, we get $\langle\omega\rangle \gg 5\times 10^{-7}\,\mathrm{yr}^{-1}$ instead of $\langle\omega\rangle \gg 2\times 10^{-6}\,\mathrm{yr}^{-1}$. We would like to emphasize that the factor $4\pi/3$ does not influence on the upper limit on the density parameter $\Omega_{\rm gw} \ll 10^{-10}$, given by (2.15) and agreeing well with predictions of all cosmological models [1]–[3] (see also [18, 22]–[24]).

The upper limit on the density parameter $\Omega_{\rm gw} \ll 10^{-10}$ is retained if $\sqrt{\langle\omega^2\rangle} \ll 1\,\mathrm{yr}^{-1}$, which is caused by the experimental fact that the shrinkage of the machine circumference lasts longer than 5 years [9].

Of course, the constraints $\langle\omega\rangle \gg 5\times 10^{-7}\,\mathrm{yr}^{-1}$ and $\sqrt{\langle\omega^2\rangle} \ll 1\,\mathrm{yr}^{-1}$ can be justified by the properties of the spectral density $S_h(\omega)$, defined by the theoretical model of the stochastic relic gravitational–wave background.

One can suppose that in the case of the validity of our explanation of the shrinkage of the machine circumference by the relic gravitational–wave background, the constraints on the averaged frequencies of the relic gravitational waves can be likely used to set "limits on a low–frequency cosmological spectrum".

For the better understanding of the mechanism of the shrinkage of the machine circumference of the storage ring, caused by the relic gravitational–wave background, we recommend readers to consult the paper by Schin Daté and Noritaka Kumagai [8], suggested a nice physical explanation of fine variations of the machine circumference of the SPring–8 storage ring induced by the tidal forces.

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